VAN KUNNEN NAAR KENNEN

BAND 2

KANT'S CONCEPTION OF ARITHMETIC

A Defence of Kant's Theory of Arithmetic against Frege's Critique

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I. Introduction

Kant is famous for his curious theory of arithmetic as outlined in his *Critique of Pure Reason*, in which he holds mathematical truths to be synthetic a priori, as determinations of space and time. In this essay, I will discuss whether Kant's claim that arithmetical truths are synthetic a priori is defensible. I will investigate this by looking into the criticisms of Kant's account put forward by Frege, who in *The Foundations of Arithmetic* argues that arithmetical truths are analytic rather than synthetic, and considering the strength of his arguments.

In section II, I will discuss why Kant holds arithmetic to be synthetic, and argue that its syntheticity is due to its dependence on sensibility. In section III, I will present Frege's direct criticisms of Kant's view on arithmetic, and consider how much of a problem they pose for Kant's account. In section IV, I will discuss the issue of the different conceptions Kant and Frege have of the analytic/synthetic distinction. This is tied to their different conceptions of logic, and in general to Kant's very narrow view on what logic consists of and what it can achieve. In section V, I will discuss what Kant's view on quantified logic would be, and introduce the problem of inequations. This problem raises the question which lies at the heart of the disagreement between Kant and Frege, namely what the objects of arithmetic are. In section VI, I will conclude that Kant's classification of arithmetic as synthetic a priori knowledge is defendable against Frege's criticisms, since his most fundamental criticism only carries weight when one ascribes to Frege's conception of the analytic/synthetic distinction. Kant's conception of the distinction is the more interesting one to ascribe to, though, since it accounts for the close connection between arithmetic and the structure of our empirical reality.

2. Kant's Theory of Arithmetic

In order to argue that Kant has good reason to view arithmetic as synthetic a priori knowledge, it is first necessary to establish whence arithmetic derives its syntheticity in Kant's account. The only thing that is clear regarding this question is that Kant thinks arithmetic has a special connection to intuition, and that this connection to intuition makes arithmetical truths synthetic. In this section, I will examine what this connection to intuition consists of. I will first discuss Hintikka's view that 'intuition' for Kant means a singular representation. On his view, arithmetic is synthetic simply because arithmetical reasonings require the use of singular representations. I will argue against this interpretation, because I believe it eliminates the most interesting aspect of Kant's account of arithmetic: its dependence on the faculty of sensibility. I will then suggest an interpretation of Kant that makes clear how arithmetic depends on sensibility, and explain how it does not contradict Hintikka's reading of Kant that the computations involved in arithmetic are what makes arithmetic synthetic. I will conclude that Hintikka is wrong in believing we can make sense of Kant's theory of arithmetic without paying respect to the involvement of sensibility.

Hintikka's interpretation of Kant's theory of arithmetic is founded on his belief that Kantian intuitions are singular terms. Kant states early on in the Transcendental Aesthetic that intuitions and concepts are different kinds of representations of our objects of experience. Intuitions belong to the faculty of sensibility, whereas concepts belong to the faculty of understanding.[1] Sensibility is a receptive faculty by means of which we are able to perceive objects as they appear to us. In the understanding, we construct concepts by means of which we can make sense of appearances. Now, Hintikka argues that the difference in kind between intuitions and concepts comes down to the difference between singular and general terms.[2] Intuitions are terms that signify an individual object, whereas concepts signify a manifold of objects. Hintikka argues that the reason Kant thinks intuitions belong to sensibility lies in his mistaken belief that we can only obtain representations that stand for an individual object by means of sense perception.[3] The textual evidence that is most important regarding Kant's theory of arithmetic is not the Transcendental Aesthetic but the discussion of mathematics at the beginning of the Doctrine of Method.[4] According to Hintikka, this is the only way to make sense of Kant's theory of arithmetic.

At the start of the Doctrine of Method, Kant argues that in mathematics one constructs concepts, which means 'to exhibit a priori the intuition corresponding to it.'[5] The required intuition is an individual object that stands for all other possible intuitions that the concept signifies. Hintikka interprets this passage, with regard to arithmetic, in the following way:[6] The construction of a concept in the proposition '7 + 5 = 12' consists of carrying out the operation of addition on the numbers 7 and 5. The intuition that corresponds to the conceptual addition of 7 and 5, which is merely the notation '7 + 5', is the computation in which one actually adds the numbers together, the result of which is the number 12. This computation is individual in the sense that it is carried out specifically on the numbers 7 and 5. However, it also stands for all other instances of addition, because the rules that govern

[1] Kant, *Critique of Pure Reason*, A19/B34
[2] Hintikka, 'III. Kantian Intuitions', pp. 342
[3] Hintikka, 'Kant on the Mathematical Method', pp. 366-367
[4] Ibid., pp. 355-356
[5] Kant, A714/B742
[6] Hintikka, 'Kant on the Mathematical Method', pp. 365

the operation of addition in general are valid regardless of the specific numbers that are added together. The computation that is involved in establishing the truth of '7 + 5 = 12' is analogous to the construction of a triangle that is involved in proving a geometrical theorem. This is what Kant means when he says one needs to 'go beyond the concepts of 7 and 5' in order to see the truth of the proposition in his famous discussion of '7 + 5 = 12'.[7]

Hintikka argues that what for Kant serves as an intuition in arithmetic, namely the computation that consists of applying the rules of addition, subtraction, etc. to the numbers involved, cannot possibly be sensible, since there is no relation between sense perception and the rules that govern computation in arithmetic.[8] In fact, Hintikka argues that Kant himself admits in the *Prolegomena* that the intuitions involved in arithmetic are not intuitions based on sense perception.[9] This is because in arithmetic the intuition does not depend on some object of experience, but instead precedes the object that is constructed by means of this intuition. An intuition can only precede its object if it contains nothing but the *form* of sensibility. Kant calls such an intuition a pure intuition.[10] According to Hintikka, in explaining the nature of a pure intuition Kant weakens the tie between intuitions and sensibility so much that there is nothing of sense perception left. For these reasons, we should take intuitions in Kant to mean nothing other than singular representations, since the involvement of sense perception in the pure intuitions that are employed in arithmetic is both problematic and unnecessary.

I believe Hintikka is right in arguing that the intuition involved in arithmetical propositions consists of the computation in which the rules of the relevant operation are applied to the numbers in question. This interpretation conforms to the passage in the *Critique* in which Kant argues that the pure intuitions used in arithmetic are schemata for their corresponding concepts. Kant argues that, for example, the schema of number is not the image of any particular number, but 'the representation of a method for representing a multitude'.[11] Computations in arithmetic can be seen as such a method for representing a multitude, since the idea of number in general is tied to the operations that can be performed over it. However, I do not see why this interpretation requires one to sever the tie between intuitions and sensibility. When one interprets the notion of pure intuition in Kant to have no link to sense perception whatsoever, one strips Kant's theory of arithmetic of its most interesting element. The notion of pure intuition as containing the form of sensibility is a fundamental aspect of Kant's transcendental project. I will briefly explain why this is the case.

In the Transcendental Aesthetic[12], Kant argues that the pure forms of sensibility in general are space and time. Without the notions of space and time, we would not be able to have experience at all. Space and time order the sense perceptions we receive, since the world of appearances is a spatiotemporal world. We cannot conceive of appearances that are not located in space, neither can we conceive of appearances that do not follow each other in a temporal order of succession. For this reason, space and time are not empirical notions; we do not derive them from experience but rather preemptively put them into experience, because without a way of ordering sensations we cannot have experience at all. This is why space and time are the a priori forms of the sensibility in general. Pure intuitions, which contain nothing but the form of sensibility, are determinations of space and time. Now, what

[7] Kant, B16
[8] Hintikka, 'Kant on the Mathematical Method', pp. 367
[9] Hintikka, 'III. Kantian Intuitions', pp. 343-344
[10] Kant, A21/B35
[11] Kant, A141/B180
[12] Kant, B33-B73

makes pure intuitions interesting is that, as determinations of space and time, they both abstract from *and* contain every possible object of sense perception, since all possible experience is spatiotemporal. This is why arithmetic, a science based on pure intuition, is both a priori and synthetic. It is a priori because pure intuitions do not depend on any particular object of sense perception, but synthetic because pure intuitions contain the form of every possible object of sense perception.

What I have given above is a general explanation of why the tie between arithmetic and sense perception is significant for Kant, but I have not answered the question Hintikka raises of how exactly the pure intuitions used in arithmetic could possibly be tied to sense perception. In order to answer this question, it is necessary to clarify the exact relation between pure intuitions and empirical intuitions (intuitions of a specific object of sense perception). I believe the following passage makes this relation clear:

''One need only take as an example the concepts of mathematics, and first, indeed, in their pure intuitions. (...) Although all these principles (...) are generated in the mind completely a priori, they would still not signify anything at all if we could not always exhibit their significance in appearances (empirical objects). Hence it is also requisite for one to make an abstract concept sensible, i.e., to display the object that corresponds to it in intuition, since without this the concept would remain (as one says) without sense, i.e., without significance. Mathematics fulfills this requirement by means of the construction of the figure, which is an appearance present to the senses (even though brought about a priori). In the same science the concept of magnitude seeks its standing and sense in number, but seeks this in turn in the fingers, in the beads of an abacus, or in strokes and points that are placed before the eyes. The concept is always generated a priori, together with the synthetic principles or formulas from such concepts; but their use and relation to supposed objects can in the end be sought nowhere but in experience, the possibility of which (as far as its form is concerned) is contained in them a priori.[13]''

In this passage Kant clarifies why arithmetic is necessarily synthetic. Since arithmetical propositions are determinations of space and time, and space and time are the forms of sensibility in general, arithmetical propositions derive their significance from the fact that they are always applicable to some object of possible experience. Now, the object that is constructed in pure intuition is not any particular object of sense perception. In this sense, Hintikka is right in arguing that the computations involved in arithmetic cannot be based on sense perception. It would be ludicrous to argue that, say, in order to know the truth of '1000 + 1000 = 2000' one needs to imagine a collection of a thousand points and another collection of a thousand points, and count how many points there are in total. However, I believe this is not what Kant is getting at when he talks about the sensible objects of fingers, points, etc. What makes arithmetical propositions universally valid is the fact that one *could* apply the computation involved in establishing their truth to any empirical intuition whatsoever.[14] This possibility is ensured by the fact that pure intuition contains the form of every particular empirical intuition.

[13] Kant, A240/B299

[14] After writing this argument, it has come to my attention that L. Shabel makes this point as well in her discussion of the pure intuition in mathematical demonstration: Shabel, *Mathematics in Kant's Critical Philosophy*, pp. 106-107

To this, Hintikka might respond that this conception of pure intuition still appears to be more intellectual than sensible. A non-empirical term can stand for a number of empirical objects, so the fact that pure intuitions contain within them a manifold of empirical intuitions does not necessarily make them sensible. My reply to this point is that pure intuitions are fundamentally sensible because the above passage shows that the truth of a proposition that is based on pure intuition depends on the structure of our perceptual experience. If this structure were not spatiotemporal, arithmetical propositions would not necessarily be true. Since the truth of arithmetical propositions depends on the structure of our sensibility, and it is pure intuition by means of which we determine the truth of a proposition in arithmetic, pure intuition must necessarily be sensible.

To summarize, we have seen that Hintikka correctly explains Kant's claim that arithmetical propositions are synthetic due to their connection to intuition. This connection to intuition means that in order to see the truth of an arithmetical proposition, one must carry out a computation in which the rules of arithmetical operations are applied to the proposition in question. This computation consists of the application of what Kant calls a schema, which is a pure intuition that serves as a general method for representing a multitude, to the concepts involved in the proposition. However, Hintikka is wrong in stating this pure intuition is not necessarily connected to the faculty of sensibility. Pure intuitions are connected to empirical intuitions because, as determinations of space and time, they contain the form of all empirical intuitions in general. Since pure intuitions by virtue of which we can establish the truth of arithmetical propositions, Kant argues that the truth of arithmetical propositions depends on pure intuition, and therefore on the form of sensibility in general. This dependence on sensibility is what makes arithmetic synthetic. In the next section, I will discuss Frege's criticisms of this account of arithmetic.

3. Frege's Critique of Kant

In the first chapter of *The Foundations of Arithmetic*, Frege brings forward three criticisms of Kant's theory of arithmetic. In this section, I will present each criticism and argue how Kant's account can be defended against them. Regarding the third criticism, I should note that the whole project of *The Foundations of Arithmetic* can be seen to constitute an argument against Kant's claim that arithmetical truths are synthetic, since this project is an attempt to ground arithmetic in the laws of logic alone. I will not be able to review the whole of this work, but Frege's third criticism of Kant's account does provide a ground for discussion of the fundamental differences between the philosophical views of Kant and Frege.

Frege's first criticism[15] concerns the notorious discussion of '7 + 5 = 12' in the Introduction in the second edition of the *Critique*:

> ''The concept of twelve is by no means already thought merely by my thinking of that unification of seven and five, and no matter how long I analyze my concept of such a possible sum I will still not find twelve in it. One must go beyond these concepts, seeking assistance in the intuition that corresponds to one of the two, one's five fingers, say, or (as in Segner's arithmetic)! five points, and one after another add the units of the five given in the intuition to the concept of seven.''[16]

According to Frege, the only method of proof Kant admits of to see if a certain addition proposition is true, is in the case of small numbers counting physical objects to see that their truth is evident, and in the case of large numbers carrying out a formal proof. Frege criticizes this theory because making a distinction between small and large numbers is an awkward matter. The distinction will always be arbitrary. Besides, proof by appeal to physical objects appears to be an empirical method, even though Kant holds arithmetic to concern pure intuition.

I believe that this criticism does not hold, since Frege puts far too much emphasis on the passage above, which gives the counting of fingers as an example of mathematics being guided by intuition. This example indeed excludes large numbers and gives the impression that arithmetic relies on empirical intuition. However, as I argued above, Kant uses the counting of fingers merely as an example of a way in which one *could* make an appeal to intuition. Arithmetical truths have meaning because of the possibility of making such an appeal to intuition, but this does not mean that an appeal to empirical intuition is necessary at every inference. The schemata that make arithmetical propositions applicable to empirical intuitions are all that is necessary in order to see the truth of these propositions.

Regarding Kant's mentions of counting physical objects throughout the *Critique,* I believe these are meant to illustrate the tie between arithmetic and experience by calling attention to the psychological origins of arithmetic. Children first learn to count by means of physical objects before they learn the rules governing addition. Of course, once these rules are internalized one can carry out additions without the aid of physical objects. Kant's distinction between seeing the truth of arithmetical propositions involving small numbers in perception and proving the truth of those involving large numbers is no more awkward than the distinction between a child carrying out addition and an adult doing the same in a

[15] Frege, *The Foundations of Arithmetic*, §5
[16] Kant, B16

different manner. So, Frege's first criticism does not hold, since it relies on a misconstruction of Kant's argument. In his second criticism[17], Frege argues against Kant's claim that arithmetic is synthetic by pointing out the dissimilarity between arithmetic and geometry. Frege argues that geometry is synthetic, since an individual geometrical figure constructed in intuition can serve to represent other figures of the same kind. This is because a geometrical point (or line, plane, etc.) is by itself indistinguishable from any other point. However, the construction of a particular number in intuition cannot represent all other numbers at the same time, since 'each number has its own peculiarities'. Constructions of individuals in arithmetic therefore cannot represent other individuals of the same kind like they can in geometry, which means arithmetic cannot be synthetic.

I believe this criticism, like the first one, comes down to a misunderstanding of Kant's theory. It should first be pointed out that Kant does think that arithmetic is unlike geometry, since for Kant arithmetic consists of determinations of space and time, whereas geometry deals only with determinations of space.[18] For this reason, in geometry it is possible for visual constructions to serve as schemata. Arithmetic, however, is a more general science, since it concerns successive addition, subtraction, etc. in general. Because arithmetic is more general than geometry, the constructions in arithmetic cannot be visual. In his discussion of the schema of number, Kant points out that this schema is a representation of a general procedure which can produce images of individual numbers, but is not an image of any number itself.[19] So, Frege is right in arguing that geometrical constructions in intuition are unlike arithmetical constructions in intuition, but Kant agrees with this. Kant does not believe the construction of a particular number in arithmetic can serve to represent other numbers, but a nonvisual method which abstracts from any peculiarities particular numbers may have and only represents them in as far as they are numbers is a construction in intuition that can serve to represent any and all numbers. It is this representation of a general method by virtue of which arithmetic is synthetic.

This brings us to Frege's third criticism[20], which concerns the generality of arithmetic. This criticism is a general objection to Kant's claim that arithmetic is synthetic a priori knowledge. Kant views arithmetical propositions to be determinations of space and time, and therefore constricted to the realm of possible experience. Frege argues that Kant cannot base arithmetic on sensibility, because the domain of arithmetic exceeds the domain of possible experience. One can imagine the wildest notions in which laws like gravity are defied, notions which are not objects of possible experience at all. However, what one cannot imagine is '2 + 2 = 5'. What we are concerned with here is not a law of possible experience but a law of thought. By tying arithmetic to sensibility Kant unfairly reduces the domain of arithmetic from everything thinkable to everything intuitable.

This criticism appears to be very strong, and by means of it Frege shows why he views arithmetic to be a part of logic: Both arithmetic and logic concern the widest domain possible, namely the domain of thought. I do not believe the criticism refutes Kant's view, though. Kant would probably reply to this criticism by arguing that the domain of thought, meaning proper cognition, is the domain of possible experience. Concepts only constitute cognition if they are thought in combination with their corresponding intuition.[21] Intuitions are always

[17] Frege, §12-13
[18] Kant, B41, A143/B182
[19] Kant, A141/B180
[20] Frege, §14
[21] Kant, B148

spatiotemporal, and therefore proper cognition is always spatiotemporal as well. However, why we should think Kant is right in claiming that cognition necessarily involves spatiotemporal intuition is not a question I can straightforwardly answer. I will first have to establish what exact distinction between analytic truths and synthetic truths Kant upholds, and what his views on logic are. I will do this in the following two sections.

4. Analyticity in Kant and Frege

In order to answer the question of whether Kant was right in arguing that mathematical truths are synthetic a priori, and whether Frege's final criticism holds, it is necessary to address the more fundamental question of which definition of the distinction between analytic and synthetic truths we should employ. In this section, I will first look into Kant's view on the distinction. All we have established so far is that arithmetical truths are synthetic because their truth depends on their applicability to experience, but we have not discussed what Kant's analytic/synthetic distinction in general consists of. There is no consensus on what Kant's exact definition of the distinction is, since his language in the *Critique* concerning the matter is rather vague. However, by juxtaposing Brittan's and Beck's differing explanations of Kant's distinction, I hope to reach a conclusion about the analytic/synthetic distinction in Kant that does justice to his transcendental project. I will then consider Frege's different conception of the distinction. By Frege's definition of the analytic/synthetic distinction, arithmetical truths are analytic. I will conclude that in order to argue that Kant's distinction is the one to uphold, we need to show that his distinction is the more interesting one. In order to do this, it will be necessary to look at Kant's view on logic.

Regarding the question of how Kant conceives of the analytic/synthetic distinction, Brittan provides a very neat explanation. Kant famously marks the difference between analytic and synthetic judgments as the difference between judgments in which the predicate inheres in the subject and judgments in which the predicate lies outside of the subject.[22] Regarding analytic judgments, all that is necessary in order to see that they are true is an analysis of the subject, to see that the predicate is part of it. This is why 'All bachelors are unmarried' is analytic, since by substitution of synonyms we get: 'All unmarried men are unmarried'. However, this (rather vague) definition of the distinction seems to work only for truths that are of a subject-predicate form. Kant also holds, though, that there are analytic principles in mathematics, such as the law of identity (a = a), which do not have the subject-predicate form.[23] Brittan accounts for this by arguing that both mathematical analytic principles and analytic judgments of the subject-predicate form can be reduced to instances of the law of contradiction. The negation of 'All bachelors are unmarried' implies that a thing can be both married and unmarried, just as the negation of the law of identity implies that a thing can be both a and not-a.[24]

Furthermore, Brittan argues that a helpful way to think about Kant's analytic/synthetic distinction is by invoking possible worlds, even though this is not a notion Kant considers himself. Since for Kant analytic truths are true 'irrespective of content', we should think of analytic truths as being true in all possible worlds, since the contents of these different possible worlds do not bear on the truth of analytic propositions.[25] However, explaining the distinction for Kant in this way seems to validate Frege's criticism of Kant's classification of arithmetical truths. Since even in my wildest imaginings I cannot conceive of '2 + 2 = 4' being false, this truth must be analytic. Brittan draws a distinction between logical possibility and 'real possibility', though. 'Real possibility' for Kant is the realm of possible experience. Since according to Kant the laws of our mental faculties restrict the kind of experience we are

[22] Kant, A7/B11
[23] Kant, B17
[24] Brittan, *Kant's Theory of Science*, pp. 16
[25] Brittan, pp. 17-18

capable of having to spatiotemporal-causal experience, the realm of possible experience (the really possible worlds) is a proper subset of the logically possible worlds. Truths that hold in all and only really possible worlds are synthetic a priori, and truths that hold in all logically possible worlds are analytic a priori.[26]

Now, it is intuitively plausible that in all possible worlds we can imagine '2 + 2 = 4' must be true, but to this Kant would in all likelihood reply that the mere act of imagining a possible world requires it to be spatiotemporal-causal. For example, even though 'Every event has a cause' is for Kant not necessarily an analytic truth (depending on one's definition of the concept 'event'), we cannot really *imagine* a world in which events do not have causes, since the objects of our imagination are tied by the same laws as the objects of possible experience. However, Frege would certainly not infer from this that the judgment 'Every event has a cause' is analytic. The important point Kant makes is that the laws of arithmetic, like the laws of physics, are only universally valid in the realm of possible experience. So, given this distinction between analytic a priori truths and synthetic a priori truths for Kant, Frege's criticism does not constitute a straightforward refutation of Kant's theory of arithmetic. Whether it can be argued, outside of Kant's system, that arithmetical truths hold not only in the realm of all really possible worlds but also in the realm of all logically possible worlds, is a question that requires further investigation.

Brittan's explanation is, as I said above, very neat and for that reason attractive. However, it is not entirely unproblematic. In passing I mentioned that for Kant, the classification of the judgment 'Every event has a cause' as analytic or synthetic depends on what definition of 'event' one employs in making the judgment. If the analyticity of a judgment depends on the definition of its concepts, though, the line between analytic and synthetic becomes blurred. The distinction is in danger of being reduced to: 'Whether a judgment is analytic or synthetic depends entirely on which definition one gives of the concepts involved', which is hopelessly trivial. Furthermore, Brittan explains the distinction in purely logical terms, whereas there is also strong textual evidence for Kant conceiving of the distinction in psychological terms.[27] Of course, for Kant it might obviously be the case that the logical notion of analyticity and the psychological notion of analyticity have the same extension, but for us this is not at all obvious. For these reasons, Beck argues that the analytic/synthetic distinction in Kant is slightly more complicated. I will now give an outline of Beck's explanation of the distinction.

Beck argues that Kant appears to employ two criteria in judging whether a proposition is analytic:[28] (1) A logical criterion: Analytic propositions conform to the law of contradiction, we can judge whether a proposition conforms to this law by means of a partial analysis of the subject concept. A full definition is not required. Kant views the full definition of a concept to be the end of analytic thought, not a prerequisite for it. Where concepts A and B in a judgment mark object X, in the case of an analytic judgment the object marked is not relevant, since the judgment expresses a logical relation between concepts A and B.[29] (2) A psychological criterion: In analytic propositions, the predicate concept is really thought in the subject concept. Beck argues that Kant mistakenly thinks the two criteria to have the same extension, when in fact Kant employs the logical criterion where definitions or analyses close

[26] Brittan, pp. 21-22
[27] Kant, A7/B11-A8/B12
[28] Beck, pp. 8-9
[29] Beck, pp. 5-6

to definitions of the concepts are available, and resorts to the psychological distinction where definitions are lacking.[30]

When it comes to the synthetic a priori propositions that follow from the Categories, though, Beck argues that Kant's position can, at least partially, be made sense of. For Kant, the indefinability of the group of concepts that constitutes the Categories is not a problem to be solved, but a fundamental characteristic of the Categories. These concepts are not fixed by definition, but fixed by virtue of the fact they are pure concepts. Their meaning is fixed because the meaning of a pure concept must be in accord with the 'general condition of sensibility', i.e. must be applicable to possible experience in general.[31] Now, mathematical concepts, as opposed to the Categories in general, are strictly definable. As Beck argues, the reason that specific mathematical concepts are not indefinable, like the Categories in general are, is because the objects of mathematics are constructed in intuition.[32]

I believe that by means of the example of mathematical concepts, it is possible to show how Kant's logical and psychological criterion really do have the same extension, since they are connected. I will only discuss arithmetic, not mathematics in general. Arithmetic is synthetic by both the logical and the psychological criterion. It is synthetic by the logical criterion, because arithmetical truths are true by virtue of the fact that their corresponding objects are constructed in intuition. The possibility of forming arithmetical propositions thus relies on the objects that the concepts involved apply to, which means arithmetical propositions cannot be analytic. Arithmetic is also synthetic by the psychological criterion, since in order to arrive at arithmetical truths, its objects first need to be constructed in intuition. The addition of the numbers 7 and 5 are therefore not really thought in the concept of the number 12, since these concepts are not enough to come to know the truth of the proposition '7 + 5 = 12'. In order to form this proposition, the corresponding objects must first have been constructed in intuition. It can be seen that in the case of arithmetic, both the logical and the psychological criterion of syntheticity come down to the correspondence of the concepts involved in the proposition to an object in intuition.

I believe this point can be generalized. For Kant, not only arithmetical truths but all synthetic truths are synthetic because the concepts involved correspond to an object in intuition. Let us look again at the example of 'All events have a cause'. If 'event' here is to mean 'a caused occurrence', the proposition is analytic. If 'event' is to mean merely an occurrence, the proposition is synthetic. However, in the Second Analogy Kant argues that we can see the truth of the second instance by observing that we need to hold it to be true in order to make sense of the empirical objects that affect us in intuition.[33] This shows why, for Kant, the logical and psychological criterion are connected: If the truth of a certain proposition does not follow from the relation between concepts and the law of contradiction alone, its truth must follow from its correspondence to objects. Due to our psychology, the only way in which objects are given to us is in intuition, therefore we need to go beyond the definitions of concepts and invoke intuition in order to see the truth of a synthetic proposition. The logical and psychological criterion have the same extension for Kant because he believes that the only kind of objects available to us are objects in intuition.

Now that some light has been shed on Kant's view on the synthetic/analytic distinction, let us look at Frege's view on the distinction. Frege's conception of the distinction

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[30] Beck, pp. 7
[31] Beck, pp. 17-18
[32] Beck, pp. 21
[33] Kant, B233-B256

is radically different from Kant's: Frege argues that psychology should not enter this distinction, the question is not how we have been able to form this proposition in consciousness (whence it originates), but rather on what ground we are justified in holding the proposition to be true.[34] If these grounds are exclusively the laws of thought, which for Frege are the laws of logic and definitions, the proposition is analytic. Now, recent efforts in the field of quantified logic have shown that it is, at least in part, possible to ground arithmetic on logical laws alone. If we therefore uphold Frege's definition of the analytic/synthetic distinction, our conclusion should be that arithmetic is analytic.

From this discussion of the analytic/synthetic distinction in Kant and Frege, it has become clear that whether arithmetic is synthetic really comes down to which of the two distinctions one employs. Here, we seem to have reached an impasse in our discussion. However, we can make a decision between the two distinctions based on which of the two is the more interesting one to uphold. This depends on the implications the distinction has regarding logical truths. Both Kant and Frege want to maintain that general logical truths are analytic, but Frege views the scope of logic to be much wider than Kant does, since Kant only considers syllogistic logic to be general logic. In order to judge which of the two philosophers holds the more interesting view on logic and its relation to arithmetic, we need to investigate whether it is significant to draw a line between syllogistic logic and other, more modern forms of logic. After all, it might be the case, as many have argued, that Kant was simply unaware of how much can be achieved using logic alone. However, simply jumping to this conclusion would be unfair to Kant, so I will investigate if there is in fact a greater significance to his analytic/synthetic distinction, outside of the fact that he needs this distinction for his entire transcendental project in the *Critique* to get off the ground.

[34] Frege, §3

5. Kant's View on Logic

The main question concerning Kant's view on logic is what he would make of modern, quantified logic. Since it has been proved that arithmetic can, at least in part, be grounded in quantified logic, speculation on the matter of what Kant would think of quantified logic is necessary. In this section, I will first argue that Kant would view quantified logic to be synthetic, using Potter's explanation of the key difference between monadic and polyadic logic. I will then explain the implications of this by discussing Brittan's argument that for Kant, to give concepts sense is to attribute truth-values to them. I will then argue that Kant's distinction between analytic monadic logic and synthetic polyadic logic has value outside of his epistemological system, using Parsons' discussion of the problem of inequations. I will conclude that the problem of inequations points towards the general reason that Kant's distinction by which arithmetic and quantified logic are classified as synthetic is thoroughly interesting: it explains the mutual conformity between arithmetical truths and reality.

With regard to the question what Kant would make of quantified logic, Potter argues that Kant would believe it is synthetic, given that Kant views analytic propositions to be true regardless of the object the proposition may apply to. Potter argues that the important difference between syllogistic logic and quantified logic concerns, not unsurprisingly, the use of quantifiers. If we consider the basic syllogistic argument 'All F's are G's and all G's are H's, therefore, all F's are H's', this argument is formulated in quantified logic in the following way:

$$\forall(x)(F(x) \rightarrow G(x))$$
$$\forall(x)(G(x) \rightarrow H(x))$$

$$\forall(x)(F(x) \rightarrow H(x))$$

It can be seen that the use of quantifiers in this argument is redundant, since the truth of the argument does not depend on them in any way. What this means is that its truth does not depend on the objects that the concepts (properties) in the argument apply to. This is why only syllogistic logic is analytic for Kant; only syllogistic arguments involve nothing other than the comparison of concepts. When quantifiers overlap in a proposition in quantified logic, the objects that concepts apply to become relevant to the truth of this proposition.[35] This is why Kant would have regarded quantified logic in general to be synthetic, except in the case of propositions which can be formulated in syllogistic logic as well. Kant therefore draws the line between monadic and polyadic logic.

What is the significance of distinguishing between monadic and polyadic logic for Kant? Brittan argues that this distinction establishes the difference between a meaningless play on concepts and proper cognition. In section II, I explained Kant's claim that without the possibility of applying arithmetical propositions to empirical intuitions these propositions are meaningless to mean that without this possibility we cannot establish that these propositions are necessarily true. Brittan generalizes this idea by arguing that, in general, propositions acquire truth-values only when they are provided with a corresponding intuition. This is because only through intuition objects of possible experience can be given to us, and the truth-value of a proposition depends on its applicability to objects of possible experience.[36]

[35] Potter, *Reason's Nearest Kin*, pp. 33-34
[36] Brittan, pp. 62

Now, this does not seem to be true, since analytic propositions, being universally true, have truth-values independent of their applicability to objects. Brittan explains the status of analytic propositions for Kant by arguing that Kant believes such propositions to be empty and therefore meaningless. They are true merely by virtue of the definitions of the concepts involved in the proposition. In order to have truth-values a proposition needs to be meaningful, and in order to be meaningful a proposition must depend on objects of possible experience. The most important point Kant makes here is of course the one I discussed in section IV, namely that objects are only given to us in intuition, and that all objects we can conceive of are therefore objects of possible experience. Note here that Kant does not claim that the only existent objects are objects of possible experience. There are objects which exist independently of our experience (noumena), but we cannot know anything about these objects outside of the fact that they exist.[37]

Now that we have established what it means for Kant to make a distinction between the epistemological value of monadic logic and polyadic logic, let us see if this distinction is interesting outside of his account. Both Kant and Frege believe that the objects of arithmetic are the same as the objects of quantified logic. Now, given the agreement between Kant and Frege on this point, there is reason to argue why Kant's distinction between monadic and polyadic logic is significant: namely the problem of inequations. I will give a general outline of Parsons' explanation of this problem.

Parsons starts his explanation by discussing the proposition '2 + 2 = 4'. This proposition can be formalized and proved in first-order logic with identity. I will here omit the technical formulation of the proposition, since it is not vital to the general explanation. The formulation of '2 + 2 = 4' in first-order logic comes down to a material implication: 'If there are two things, and there are another two things, then there are four things'. This proposition is valid in all interpretations. However, now consider '2 + 2 = 5', again formalized in first-order logic with identity. This proposition is false in some interpretations, but not in all of them. In a possible world in which there are only three objects or less, this proposition is as true as '2 + 2 = 4', since the antecedent of the material implication (which is the same in both propositions) comes out as false. Therefore, we can put anything we like in the place of the consequent, since the implication will always be true. From this we can see that in order to ensure that inequations like '2 + 2 ≠ 5' are universally true, we need to make certain existence assumptions.[38]

The problem of inequations forms an argument for Kant's claim that the objects of arithmetic and quantified logic are the objects of possible experience. If we restrict the interpretations of '2 + 2 = 5' to really possible worlds, this proposition will be false in all of them, since the spatiotemporal-causal form of our experience requires there to exist an infinite amount of objects. We can intellectually postulate a world in which there are only a finite number of objects, but due to the fact that space and time are both infinitely large and non-empty, we cannot properly imagine such a world, because the corresponding spatiotemporal intuition is lacking. This means that the world in question does not belong to the realm of really possible worlds. The existence axioms required to establish the universal truth of all inequations are thus provided by the spatiotemporal-causal form of objects of possible experience. Kant can therefore account for inequations in a way that Frege cannot. If the laws of arithmetic are as general as the laws of logic, the inequation '2 + 2 ≠ 5' should be true in all logically possible worlds. However, we have seen that it is not.

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[37] Kant, A256/B312

[38] Parsons, 'Kant's Philosophy of Arithmetic', pp. 28-29

There might be a way for Frege to defend his classification of arithmetic as analytic knowledge in the face of the problems of inequations. For Frege, the realm of objects of arithmetic and quantified logic is not restricted by the structure of our sensibility, but it is restricted by the structure of our thought.[39] The laws of arithmetic are valid in the domain of objects we can construct in thought, and our understanding is capable of constructing the representation of an infinite number of objects. Since an infinite number of objects is the widest domain possible, the truth of inequations in this domain establishes their necessity, since what is true for every object is necessarily true.

However, this defence requires Frege to replace the logical notion of necessary truth as truth in all interpretations with a notion of necessary truth that appears to be metaphysical rather than logical, namely truth in the broadest domain possible. The connection between arithmetic and logic is lost in this explanation. Furthermore, the appeal to construction of a representation of an infinite number of objects in thought comes dangerously close to Kant's idea of construction in intuition, while Frege wishes to avoid any such psychological basis for the truth of arithmetic. This possible defence against the problem of inequations therefore does not hold, since it creates more problems for Frege's account than it solves.

Now, the problem of inequations by itself is of course not enough reason to argue that Kant's conception of the analytic/synthetic distinction is more interesting than Frege's. One could argue that logicians have not yet been able to ground inequations in quantified logic, but this might very well be achieved in the future, since the grounding of arithmetic in logic is an ongoing process. However, I believe the problem of inequations signifies a more general argument for Kant's analytic/synthetic distinction. Kant's idea that the validity of arithmetical truths is restricted to the realm of possible experience accounts not only for the fact that these truths are applicable to all empirical objects, but also for the fact that our empirical reality in general seems to conform to arithmetical truths. In the case of the problem of inequations, this is the case because the infinity of space and time excludes the real possibility of a world with only three or less objects in it. The application of the science of arithmetic advances empirical sciences such as physics, but in turn empirical sciences advance the science of arithmetic, for example when physics maps a natural law by expressing it in a particular algebraic proposition, which turns out to be significant for algebra (and by extension arithmetic) in general. The sciences of arithmetic and quantified logic are dynamic sciences, because their connection to possible experience allows them to be expanded and advanced.

The fact that Kant's theory of arithmetic accounts for this mutual correspondence between the laws of arithmetic and the objects of possible experience is why his conception of the analytic/synthetic distinction is in my view more interesting. Frege's conception of the distinction does not account for this peculiarity of arithmetic, since it does not allow for any fundamental distinction between arithmetic as grounded in polyadic logic and other forms of logic. Both depend solely on the laws of thought.

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[39] Potter, pp. 68

6. Conclusion

In this essay, I set out to answer the question of whether Kant's claim that arithmetic is synthetic a priori knowledge is defensible against Frege's criticisms. In section II, I have argued that arithmetic is synthetic in Kant's theory because the truth of arithmetical propositions depends on the general structure of our faculty of sensibility. In section III, I discussed Frege's three criticisms of Kant's theory of arithmetic. I have shown how the first two criticisms are directly refutable because the arguments are based on misunderstandings of Kant's theory. The third criticism I could not straightforwardly answer. It was first necessary to explain the differences between Kant's and Frege's conception of the analytic/synthetic distinction. I discussed these two conceptions in section IV. Here, I have argued that for Kant, the distinction between analyticity and syntheticity is the distinction between propositions that are true by virtue of the definitions of concepts involved, and propositions which derive their truth from their correspondence to an object in intuition. For Frege, the distinction is that between propositions whose truth is established by the laws of logic and definitions, and those whose truth is established by other means. Since the status of arithmetical truths depends on which distinction one upholds, it was necessary to find out which is the more interesting one. In section V, I answered this question by showing that for Kant, both arithmetic and quantified logic are synthetic, because their truth depends on their application to objects of experience. This conception of syntheticity is interesting because it not only solves the problem of inequations, but also in general accounts for the mutual correspondence between arithmetic and quantified logic on the one hand, and empirical reality on the other.

So, to answer the question of whether Kant's theory of arithmetic is defendable against Frege's criticisms, the answer is positive, because Frege's third objection loses its power in the face of the problem of inequations, and because, more significantly, in claiming that arithmetic is synthetic, Kant accounts for the close connection between arithmetic and empirical reality in a way that Frege cannot. This is because Frege's own theory throws the truth of propositions like '7 + 5 = 12' and 'All bachelors are unmarried' together on one large heap of analyticity.

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